RELIABILITY EVALUATION USING MAGDM BASED ON TRIANGULAR INTUITIONISTIC ATTITUDINAL RANKING AND AGGREGATING MODEL

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Abstract. This paper presents a Triangular intuitionistic attitudinal ranking and aggregating (TIARA) model for reliability evaluation and prediction, at the conceptual stage of new product development. The TIARA model which is based on an Induced triangular intuitionistic hybrid fuzzy weighted geometric (I-TIHFWG) operator and an attitudinal rank score function, has the following advantages over currently existing ones. It's account for the attitudinal character of design and reliability assessment experts in the evaluation and prediction of reliability at the early product design stage and for decision-making. It's reduces the complexity in the product development process by representing holistically all the complexity and uncertainty using the Triangular intuitionistic fuzzy number (TIFN) which is a more generalized platform for expressing imprecise and inconsistent information and finally. It provides an opportunity for carrying out a sensitivity analysis using the attitudinal score function (attitudinal parameter), thereby addressing the ranking problem normally associated with the TIFN(s). To demonstrate the effectiveness, feasibility, and rationality of the proposed model, it was applied for the evaluation of a hypothetical reliability assessment problem in literature and has been compared with the similar computational model. In the future, we will continue working on the application of the proposed model in other domain.

1. Introduction

Reliability which has become a default requirement in the design of today's products, mostly for complex products and systems, are intended mainly to help in the identification of components that need modification early in the product design stage, improve the product safety, increases its serviceability when in operation and reduces its future maintenance costs (Majeske *et al.,* 2003). With the increased sources of product information and the high-value customers are placing on quality and the reliability of new products today (Kostina, M., 2012), the manner in which product reliability concerns are assessed and addressed has become a major concern for researchers and practitioners. According to Yang et al., (Yang *et al.,* 2011) and Smith et al., (Smith *et al.*, 2012), to improve and predict the reliability and quality of new product components during the early design phase. The failure or potential failure information of existing predecessor product, and their components should be analyzed and the information (auxiliary-based information) converted to appropriate design reliability knowledge-based decision for the new product. Since reliability information are scared at the early product design stage (Aikhuele & Turan, (a) 2016), (Sanchez, 2014) gaining such failure information from existing product component is critical to achieving an

improved decision about the new product design quality and reliability early at the product design stage (Sanchez, 2014; He, 2016; Aikhuele *et al*., 2016).

The reliability in this study, can be described as the identification of failure causes that have undesirable or significant effects on the to-be-design product; the determination of the failure modes that may seriously affect the expected or required product quality and reliability (Aikhuele & Turan, 2018). The identification of safety hazard and reliability problem areas, or the non-compliance with product safety and quality regulations, and finally, is about giving designers the opportunity to focus on areas of greatest reliability needs during the early product design stage (Group, 2007).

The evaluation and prediction of the reliability of new product during the conceptual design stage haveremained a very challenging task since the available knowledge and information at this stage are limited, descriptive and sometimes qualitative in nature (Yontay *et al.,* 2011). The traditional reliability analysis methods which are normally used and achieved through extensive testing and the use of techniques such as probabilistic modeling has been found to be inadequate in handling the uncertainty of failure data and modeling(Aikhuele & Turan, 2018; Mahapatra & Roy, 2009). Moreover, these methods tend to focused on reliability issues at the later stages of the product development and lifecycle that is the manufacturing stage, operational-testing stage, and the product maintenance when in full operation (Lu *et al.,* 1999). Recently, some new approaches had shown potentials to handling these challenges like the Fuzzy Analytical Hierarchy Process and VIKOR model for failure detection in a marine diesel engine by Balin, et al.(Balin *et al*., 2015). Euclidean distance-based similarity measure and an incremental learning clustering model presented by Tay, et al. (Tay *et al*., 2015). Fuzzy evidential reasoning and belief rule-based methodology for prioritizing failures in FMEA by Liu, et al.(Liu *et al*., 2013), and the mathematical model which is based on data envelopment analysis for analyzing the operational risk of flexible subsea risers and pipelines used for the transportation of oil and gas products by Netto, et al. (Netto *et al*., 2013). This methods and approaches, however, doesn't take into account the design stakeholder's and reliability experts behaviors (attitudinal character) in the final decision-making process, which is critical in the case of engineering projects (Merigó *et al*., 2010; Chen *et al*., 2012).

In this study, an alternative method, the Triangular intuitionistic attitudinal ranking and aggregating (TIARA) model which is based on an Induced triangular intuitionistic hybrid fuzzy weighted geometric (I-TIHFWG) operator, and an attitudinal ranking score function is proposed for the reliability evaluation and prediction, at the conceptual stage of new product development. The main advantage of this new model is that it takes into account the attitudinal character of the group of experts associated with the reliability evaluation and prediction. As well as, reduces the complexity of the product development process by representing adequately all such complexity and uncertainty in a holistic manner using Triangular intuitionistic fuzzy number (TIFN) which is a more generalized platform for expressing imprecise, incomplete and inconsistent information (Li *et al*., 2010). Furthermore, a ranking sensitivity analysis of the attitudinal score function with respect to the attitudinal parameter is provided to address the ranking problem associated with the TIFN(s) (Prakash *et al*., 2016).

In ranking TIFN, Li, (Li, 2010) introduced the score and accuracy function which to date is the most widely used method, and for converting TIFN into representative crisp value and for performing their comparison. These functions, however, cannot handle or account for the attitudinal character of experts, since it assumes the attitudinal

character of each expert is neutral. Hence, it is unable to capture holistically all the information contained and associated with the TIFN.

The rest of this paper is organized as follows. The attitudinal ranking score function for the TIFNs and a sensitivity analysis with respect to the attitudinal parameter is presented in Section 2. In Section 3, the I-TIHFWG operator is developed, and their most important properties are explored. Based on the attitudinal ranking score function and the developed operator model, in Section 4 they are investigated for solve multi-attribute decision-making problems in which the attribute are known. In Section 5, an illustrative is provided to verify the proposed model. Finally, in Section 6 some concluding remarks are presented.

2.   TIFN and the attitudinal ranking score function

To define the fuzzy nature and complexity of the real world more comprehensively, Atanassov, (Atanassov, 1986) introduced IFS, which is an extension of the traditional fuzzy set.

Definition 1. (Atanassov, 1986).

If the IFS A in $X = \{x\}$ is defined in the form,
$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\} \tag{1}$$
where $\mu_A: X \rightarrow [0,1]$, is the membership function and $v_A: X \rightarrow [0,1]$ the non-membership function, with the condition $0 \leq \mu_A(x) + v_A(x) \leq 1, \mu_A(x), v_A(x) \in [0,1], \forall x \in X$.

For each A in X, we can compute the intuitionistic index of the element x in the set A, which is defined as follows:
$$\pi_A(x) = 1 - (\mu_A(x) + v_A(x)) \tag{2}$$

Definition 2. (Aikhuele & Turan, 2017; Aikhuele & Turan, (b) 2016).

If the IFS A in $X = \{x\}$ is defined fully in the form $A = \{\langle x, \mu_A(x), v_A(x), \pi_A(x)\rangle | x \in X\}$, where $\mu_A: X \rightarrow [0,1]$, $v_A: X \rightarrow [0,1]$ and $\pi_A: X \rightarrow [0,1]$. The different relations and operations for the IFS are;
1. $A.B = \{\langle x, \mu_A(x).\mu_B(x), v_A(x) + v_B(x) - v_A(x).v_B(x)\rangle | x \in X\}$;
2. $A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), v_A(x).v_B(x)\rangle | x \in X\}$;
3. $\lambda A = \{\langle x, 1 - (1 - \mu_A(x))^\lambda, (v_A(x))^\lambda\rangle | x \in X\}$, $\lambda > 0$;
4. $A^\lambda = \{\langle x, (\mu_A(x))^\lambda, 1 - (1 - v_A(x))^\lambda\rangle | x \in X\}$, $\lambda > 0$;
5. $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $v_A(x) = v_B(x)$ for all $x \in X$;
6. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$.

Recently, the IFSs have been extended to the Triangular Intuitionistic Fuzzy Sets (TIFSs), with the characteristic membership and non-membership values represented with the TIFN (Li, 2010). The TIFN is therefore denoted as $\acute{\alpha} = \langle ([a, b, c]; \mu_\alpha), ([a, b, c]; v_\alpha)\rangle$, when $\mu_\alpha = 1$, and $v_\alpha = 0$, $\acute{\alpha}$ will change into the traditional triangular fuzzy number (TFN). Generally the TIFN $\acute{\alpha}$ is defined as $\acute{\alpha} = ([a, b, c]; \mu_\alpha, v_\alpha)$ for conveniences, with the membership function represented as:

$$\mu_\alpha(x) = \begin{cases} \frac{(x-a)\mu_\alpha}{(b-a)} & (a \le x < b), \\ \mu_\alpha & (x = b) \\ \frac{(c-x)\mu_\alpha}{c-b} & (b < x \le c), \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and the non-membership function as;

$$v_\alpha(x) = \begin{cases} \frac{(b-x+v_\alpha(x-\acute{a}))}{(b-\acute{a})} & (\acute{a} \le x < b), \\ v_\alpha & (x = b) \\ \frac{(x-b+v_\alpha(\acute{c}-x))}{\acute{c}-b} & (b < x \le \acute{c}), \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $0 \le \mu_\alpha \le 1; 0 \le v_\alpha \le 1; 0 \le \mu_\alpha + v_\alpha \le 1, a, b, c, \acute{a}, \acute{c} \in \mathbb{R}$.

Definition 3. (Zhang & Liu, 2010; Liang *et al.*, 2014)

Let $\alpha_1 = ([a_1, b_1, c_1]; \mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = ([a_2, b_2, c_2]; \mu_{\alpha_2}, v_{\alpha_2})$ be two TIFN and $\lambda \le 0$ then. The operational results for the two TIFNs are given in the theorem.

1. $\alpha_1 + \alpha_2 = \alpha_2 + \alpha_1$
2. $\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1$
3. $\lambda(\alpha_1 + \alpha_2) = \lambda\alpha_1 + \lambda\alpha_2 \lambda \ge 0,$
4. $\lambda_1\alpha + \lambda_2\alpha = (\lambda_1 + \lambda_2)\alpha \; \lambda_1\lambda_2 \ge 0$
5. $\alpha^{\lambda_1} \otimes \alpha^{\lambda_2} = \alpha^{\lambda_1+\lambda_2} \lambda_1\lambda_2 \ge 0$
6. $\alpha_1{}^\lambda \otimes \alpha_2{}^\lambda = (\alpha_1 \otimes \alpha_2)^\lambda \lambda \ge 0$

Definition 4. (Wan *et al.*, 2017).

Let $\alpha_1 = ([a_1, b_1, c_1]; \mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = ([a_2, b_2, c_2]; \mu_{\alpha_2}, v_{\alpha_2})$ be two TIFN, the Hamming distance between α_1 and α_2 is given as;

$$d(\alpha_1, \alpha_2)$$
$$= \frac{1}{6}\left[\begin{array}{c} \left|(1 + \mu_{\alpha_1} - v_{\alpha_1})a_1 - (1 + \mu_{\alpha_2} - v_{\alpha_2})a_2\right| + \left|(1 + \mu_{\alpha_1} - v_{\alpha_1})b_1 - (1 + \mu_{\alpha_2} - v_{\alpha_2})b_2\right| \\ + \left|(1 + \mu_{\alpha_1} - v_{\alpha_1})c_1 - (1 + \mu_{\alpha_2} - v_{\alpha_2})c_2\right| \end{array} \right] \tag{5}$$

To rank the TIFN, Li, (Li, 2010)introduced the score and accuracy function which has become the most widely used method for ranking TIFNs, for converting TIFN into representative crisp value and for performing their comparison.

Definition 5. (Li, 2010).

Let $\acute{\alpha} = ([a, b, c]; \mu_\alpha, v_\alpha)$ be a TIFN. If the membership and non-membership functions are represented by the score function $S(\acute{\alpha})$ and accuracy function $H(\acute{\alpha})$ respectively, then $\acute{\alpha}$ can be defined as follow;

$$S(\acute{\alpha}) = \frac{(a+2b+c)\mu_\alpha}{4} \tag{6}$$

$$H(\acute{\alpha}) = \frac{(a+2b+c)(1-v_\alpha)}{4} \tag{7}$$

Let $\acute{\alpha}_1$ and $\acute{\alpha}_2$ be two TIFN. If $S(\acute{\alpha}_i) = \frac{(a_i + 2b_i + c_i)\mu_{\alpha_i}}{4}$ and $H(\acute{\alpha}) = \frac{(a_i + 2b_i + c_i)(1 - v_{\alpha_i})}{4}$ are the membership and non-membership functions of $\acute{\alpha}$ then;

1. If $S(\acute{\alpha}_1) < S(\acute{\alpha}_2)$ then $\acute{\alpha}_1 < \acute{\alpha}_2$
2. If $S(\acute{\alpha}_1) = S(\acute{\alpha}_2)$ and $H(\acute{\alpha}_1) = H(\acute{\alpha}_2)$, then $\acute{\alpha}_1 = \acute{\alpha}_2$
3. If $S(\acute{\alpha}_1) = S(\acute{\alpha}_2)$ and $H(\acute{\alpha}_1) < H(\acute{\alpha}_2)$, then $\acute{\alpha}_1 < \acute{\alpha}_2$

Although the above score and accuracy functions are effective in converting TIFN into representative crisp value and for performing their comparison, however, they are unable to take into account the design stakeholder's and reliability experts attitudinal character which is critical in the evaluation and prediction of reliability in an engineering design project. To overcome this shortcoming, the attitudinal ranking functions are developed from the traditional Li's score and accuracy functions.

Definition 6. Let $\acute{\alpha} = ([a, b, c]; \mu_\alpha, v_\alpha)$ be a TIFN. If the membership and non-membership functions are represented by the attitudinal score function $AS(\acute{\alpha})$ and attitudinal accuracy function $AH(\acute{\alpha})$ respectively, and then $\acute{\alpha}$ can be defined as follow;

$$AS(\acute{\alpha}) = (\lambda)\frac{(a + 2b + c + e^\lambda * \mu_\alpha{}^\lambda)}{4} \tag{8}$$

$$AH(\acute{\alpha}) = (\lambda)\frac{(a + 2b + c)(1 - v_\alpha)}{4} \tag{9}$$

where λ is the attitudinal parameter of the ranking function.

Example 1. Let $\acute{\alpha} = ([0.2, 0.3, 0.5]; 0.3, 0.45)$ and $\acute{\alpha}_1 = ([0.25, 0.3, 0.45]; 0.4, 0.5)$ be two triangular intuitionistic fuzzy set for two alternatives, then we select the desirable alternative in accordance with the attitudinal score and accuracy function, when the attitudinal parameter value $\lambda = 0.5$.

Using Eq. (8) & (9), $AS(\acute{\alpha}) = 0.275, AH(\acute{\alpha}) = 0.036$ while $AS(\acute{\alpha}_1) = 0.293$, $AH(\acute{\alpha}_1) = 0.033$, clearly from the ranking order depending on the expert's attitudinal character and the operational properties in Definition 5 it follows that; $\acute{\alpha} < \acute{\alpha}_1$. Hence, the study can conclude that the attitudinal ranking functions are able to characterize quantitatively the relations between the aggregated arguments.

3. Some TIFNs aggregation operators

Based on the attitudinal ranking function of TIFNs, we present the TIHFWG operator and develop the induced TIHFWG (I-TIHFWG) operator. Finally, we study its desirable properties. The definition of TIHFWG operator is given as follows;

Definition 7. (Liang *et al.*, 2014; Aikhuele & Odofin, 2017).

Let $\alpha_i = ([a_i, b_i, c_i]; \mu_{\alpha_{\sigma i}}, v_{\alpha_{\sigma i}})$ for all $(i = 1, 2, 3, \ldots, n)$ be a collection of Triangular Intuitionistic Fuzzy Numbers on *X*. The triangular intuitionistic hybrid fuzzy weighted geometric (TIHFWG) operator of dimension *n* is a mapping TIHFWG: $\Omega^n \to \Omega$, and associated with the weighting vector $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)^T$ to it, such that $\omega_i \in [0, 1]$, $\sum_{i=1}^n \omega_i = 1$, and is defined to aggregate a collection of intuitionistic fuzzy values $(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n)$.

$$\text{TIHFWG}_\omega(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n)$$
$$= \omega_1(\alpha_{\sigma 1}) \otimes \omega_2(\alpha_{\sigma 2}) \otimes \omega_3(\alpha_{\sigma 3}) \ldots \otimes \omega_n(\alpha_{\sigma n})$$

$$= \left(\left[\prod_{i=1}^{n}(a_{\sigma i})^{\omega_i}, \prod_{i=1}^{n}(b_{\sigma i})^{\omega_i}, \prod_{i=1}^{n}(c_{\sigma i})^{\omega_i} \right]; \prod_{i=1}^{n}(\mu_{\alpha_{\sigma i}})^{\omega_i}, 1 - \prod_{i=1}^{n}(1 - v_{\alpha_{\sigma i}})^{\omega_i} \right) \quad (10)$$

where $a_{\sigma i}$ is the i-th largest of the of α_i. Especially, $\omega = \left(\frac{1}{n}, \frac{1}{n}, ,,, 1/n \right)^T$, then the TIHFWG operator is reduced to the IHFWG operator.

The TIHFWG operator which is able to weights the intuitionistic fuzzy values, however, fails in weighting the induced ordering positions of the intuitionistic fuzzy values, in order to overcome this limitation, we develop the I-TIHFWG operator which is able to weights both the given intuitionistic fuzzy value and its induced ordering position.

Definition 8. An induced triangular intuitionistic hybrid fuzzy weighted geometric (I-TIHFWG) operator is defined as follows;

$$I - \text{TIHFWG}_{\omega,W}(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \langle x_3, \alpha_3 \rangle, \dots, \langle x_n, \alpha_n \rangle)$$
$$= \omega_1(\alpha_{\sigma 1}) \otimes \omega_2(\alpha_{\sigma 2}) \otimes \omega_3(\alpha_{\sigma 3}) \dots \otimes \omega_n(\alpha_{\sigma n})$$

$$= \left(\left[\prod_{i=1}^{n}(a_{\sigma i})^{\omega_i}, \prod_{i=1}^{n}(b_{\sigma i})^{\omega_i}, \prod_{i=1}^{n}(c_{\sigma i})^{\omega_i} \right]; \prod_{i=1}^{n}(\mu_{\alpha_{\sigma i}})^{\omega_i}, 1 - \prod_{i=1}^{n}(1 - v_{\alpha_{\sigma i}})^{\omega_i} \right) \quad (11)$$

where $a_{\sigma i}$ is the weighted intuitionistic fuzzy value $\alpha_i (\alpha_i = nw_i\alpha_i, j = 1,2,3, \dots, n))$ of the TIHFWG pair $\langle x_1, \alpha_1 \rangle$ having the i-th largest $x_i (x_i \in [0, 1])$, and x_i in $\langle x_i, \alpha_i \rangle$ is referred to as the order inducing variable and α_i is the intuitionistic fuzzy argument variable. $w = (w_1, w_2, w_3, \dots, w_n)^T$ is the weighting vector such that $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1, i = 1,2,3, \dots, n, \omega = (\omega_1, \omega_2, \omega_3, \dots, \omega_n)^T$ is the weighting vector associated with the I-TIHFWG operator with $\omega_i \in [0, 1], \sum_{i=1}^{n} \omega_i = 1$.

Theorem 1. The TIHFWG operator is a special case of the I-TIHFWG operator.

Proof: Let $\omega = \left(\frac{1}{n}, \frac{1}{n}, ,,, 1/n \right)^T$ and then

$$I - \text{TIHFWG}_{\omega,W}(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \langle x_3, \alpha_3 \rangle, \dots, \langle x_n, \alpha_n \rangle)$$
$$= \omega_1(\alpha_{\sigma 1}) \otimes \omega_2(\alpha_{\sigma 2}) \otimes \omega_3(\alpha_{\sigma 3}) \dots \otimes \omega_n(\alpha_{\sigma n})$$

$$= \frac{i}{n}(\alpha_{\sigma 1} \otimes \alpha_{\sigma 2} \otimes \alpha_{\sigma 3} \dots \otimes \alpha_{\sigma n})$$

$$= \omega_1\alpha_1 \otimes \omega_2\alpha_2 \otimes \omega_3\alpha_3 \dots \otimes \omega_n\alpha_n$$

$$= \text{TIHFWG}_\omega(\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)$$

The I-TIHFWG operator has some desirable properties, which are similar to those of the TIHFWG operator in (Liang *et al*., 2014) and they are given as follows;

Theorem 2. *Commutative property*

Let $a_{\sigma i} = ([a_{\sigma i}, b_{\sigma i}, c_{\sigma i}]; \mu_{\sigma i}, v_{\sigma i})$ and $\acute{a}_{\sigma i} = ([\acute{a}_{\sigma i}, \acute{b}_{\sigma i}, \acute{c}_{\sigma i}]; \acute{\mu}_{\sigma i}, \acute{v}_{\sigma i})(i = 1,2,3, \dots, n)$ be two triangular intuitionistic fuzzy numbers (TIFN) on *X*.

If $(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle)$ is any permutation of $(\langle x_1, \acute{\alpha}_1 \rangle, \langle x_2, \acute{\alpha}_2 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle)$

Proof: Let $\alpha_i = \acute{\alpha}_i$ for all i, that is

$$\text{I} - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle) = \prod_{i=1}^{n}(a_{\sigma i})^{w_i}$$

$$I - \text{TIHFWG}_w(\langle x_1, \acute{\alpha}_1 \rangle, \langle x_2, \acute{\alpha}_2 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle) = \prod_{i=1}^{n}(\acute{\alpha}_{\sigma i})^{w_i}$$

Since $(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle)$ is any permutation of $(\langle x_1, \acute{\alpha}_1 \rangle, \langle x_2, \acute{\alpha}_2 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle),$ we have $(a_{\sigma i}) = (\acute{\alpha}_{\sigma i}), (i = 1,2,3, \dots, n)$.
That is;

$$\left([a_{\sigma i}, b_{\sigma i}, c_{\sigma i}]; \mu_{\alpha_{\sigma i}}, v_{\alpha_{\sigma i}}\right) = \left([\acute{a}_{\sigma i}, \acute{b}_{\sigma i}, \acute{c}_{\sigma i}]; \acute{\mu}_{\alpha_{\sigma i}}, \acute{v}_{\alpha_{\sigma i}}\right).$$

Then,

$$I - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle)$$
$$= I - \text{TIHFWG}_w(\langle x_1, \acute{\alpha}_1 \rangle, \langle x_2, \acute{\alpha}_2 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle)$$

Theorem 3. *Idempotent property*

If $\alpha_i(\alpha_i - ([a_i, b_i, c_i]; \mu_i v_i) = \alpha(\alpha - ([a, b, c]; \mu, v))$ for all i, then

$$I - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle) = \alpha$$

Proof: Let $\alpha_i = \alpha$ for all i we have

$$I - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle) = \prod_{i=1}^{n}(a)^{w_i}$$

$$= \left(\left[\prod_{i=1}^{n}(a)^{w_i}, \prod_{i=1}^{n}(b)^{w_i}, \prod_{i=1}^{n}(c)^{w_i}\right]; \prod_{i=1}^{n}(\mu)^{w_i}, 1 - \prod_{i=1}^{n}(1-v)^{w_i}\right)$$

$$= \left(\left[a^{\sum_{i=1}^{n} w_i}, b^{\sum_{i=1}^{n} w_i}, c^{\sum_{i=1}^{n} w_i}\right]; \mu^{\sum_{i=1}^{n} w_i}, 1 - (1 - v^{\sum_{i=1}^{n} w_i})\right)$$

This can be rewritten as;

$$= ([a, b, c]; \mu, v) = \alpha$$

Theorem 4. *Monotonicity property*

If $\alpha_i(\alpha_i - ([a_i, b_i, c_i]; \mu_i v_i) \le \acute{\alpha}_i(\acute{\alpha}_i - ([\acute{a}_i, \acute{b}_i, \acute{c}_i]; \acute{\mu}_i \acute{v}_i),$
Then,

$$I - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \dots, \langle x_n, \alpha_n \rangle) \le I - \text{TIHFWG}_w(\langle x_1, \acute{\alpha}_1 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle)$$

Proof: Let $\alpha_i \le \acute{\alpha}_i$ for all $i,$ we have;

$$I - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle) = \prod_{i=1}^{n}(a_{\sigma i})^{w_i}$$

$$I - \text{TIHFWG}_w(\langle x_1, \acute{\alpha}_1 \rangle, \langle x_2, \acute{\alpha}_2 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle) = \prod_{i=1}^{n}(\acute{\alpha}_{\sigma i})^{w_i}$$

Since $\alpha_i \le \acute{\alpha}_i$ for all $i,$ it follows that $a_{\sigma i} \le \acute{\alpha}_{\sigma i}, (i = 1,2,3, \dots, n),$ then

$$I - \text{TIHFWG}_w(\langle x_1, \alpha_1 \rangle, \langle x_2, \alpha_2 \rangle, \dots, \langle x_n, \alpha_n \rangle)$$
$$\le I - \text{TIHFWG}_w(\langle x_1, \acute{\alpha}_1 \rangle, \langle x_2, \acute{\alpha}_2 \rangle, \dots, \langle x_n, \acute{\alpha}_n \rangle)$$

4. An approach to MAGDM Problems with TIFNs

Let's consider an MAGDM problem where a set of alternatives $A = \{A_1, A_2, A_3, \dots, A_m\}$, are assessed with respect to the criteria (attribute) denoted by $C =$

$\{C_1, C_2, C_3, \dots, C_m\}$. If the characteristics of the alternative A_i with respect to a criterion C_j are defined with a TIFN, which represents the membership, non-membership and hesitancy degree of the alternative $A_i \in A$ with respect to the criterion $C_j \in C$ for the intuitionistic fuzzy concept. The motivation here is to select the best alternative according to the intuitionistic fuzzy decision matrix $R^k(\alpha_{ij})$ $(k = 1,2,3, \dots, l)$ when the attribute weights are known.

The algorithm of proposed approach for solving the MAGDM problems when the values are expressed in TIFN is given in the following steps below.

Step 1: Set up a group of Decision Makers (DMs) $DM^k (k = 1,2,3, \dots, l)$ to express their individual evaluation or preference to the set of alternatives $A = \{A_1, A_2, A_3, \dots, A_m\}(i = 1,2, \dots, m)$ with respect to the criteria $C = \{C_1, C_2, C_3, \dots, C_m\}(j = 1,2, \dots, n)$ to obtain the intuitionistic fuzzy decision matrix $R^k = (r^k{}_i)_{mxn}$ as follows;

$$R^k = \begin{bmatrix} ([a_{11}, b_{11}, c_{11}]; \mu_{11}, v_{11}) & \cdots & \cdots & ([a_{1n}, b_{1n}, c_{1n}]; \mu_{1n}, v_{1n}) \\ ([a_{21}, b_{21}, c_{21}]; \mu_{21}, v_{21}) & \cdots & \cdots & ([a_{2n}, b_{2n}, c_{2n}]; \mu_{2n}, v_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ ([a_{m1}, b_{m1}, c_{m1}]; \mu_{m1}, v_{m1}) & \cdots & \cdots & ([a_{mn}, b_{mn}, c_{mn}]; \mu_{mn}, v_{mn}) \end{bmatrix}$$

Step 2: Using the decision information given in matrix R^k, the TIHFWG operator is used to aggregate all the decision matrices $R^k (k = 1,2,3, \dots, l)$ into a collective decision matrix $R = (r_{ij})_{mxn}$.

$$R = \left([r^k{}_i, r^k{}_i, r^k{}_i]; \mu_{r^k{}_i}, v_{r^k{}_i}\right) = TIHFWG_\omega(\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)$$

$$= \left(\left[\prod_{i=1}^n (r^k{}_i)^{w_i}, \prod_{i=1}^n (r^k{}_i)^{w_i}, \prod_{i=1}^n (r^k{}_i)^{w_i}\right]; \prod_{i=1}^n (\mu_{r^k{}_i})^{w_i}, 1 - \prod_{i=1}^n (1 - v_{r^k{}_i})^{w_i}\right)$$

where $w = (w_1, w_2, w_3, \dots, w_n)^T$ is the weighting vector of the DMs.

Step 3: Using the decision information given in matrix $R = (r_{ij})_{mxn}$, the I-TIHFWG operator is utilize to derive the overall preference values $r_i (i = 1,2,3, \dots, l)$, which is the collective comprehensive value r_i of alternative A_i.

$$r_i = \left([r^k{}_i, r^k{}_i, r^k{}_i]; \mu_{r^k{}_i}, v_{r^k{}_i}\right) = I - TIHFWG_w(\langle x_1, \alpha_1\rangle, \langle x_2, \alpha_2\rangle, \dots, \langle x_n, \alpha_n\rangle)$$

$$= \left(\left[\prod_{i=1}^n (r^k{}_i)^{w_i}, \prod_{i=1}^n (r^k{}_i)^{w_i}, \prod_{i=1}^n (r^k{}_i)^{w_i}\right]; \prod_{i=1}^n (\mu_{r^k{}_i})^{w_i}, 1 - \prod_{i=1}^n (1 - v_{r^k{}_i})^{w_i}\right)$$

where $w = (w_1, w_2, w_3, \dots, w_n)^T$ is the weighting vector of the attributes.

Step 4: Calculate the attitudinal scores function $AS(r_i)$ $(i = 1,2, \dots, n)$ and attitudinal accuracy function $AH(r_i)(i = 1,2, \dots, n)$ for the membership and non-membership functions.

$$AS(\acute{\alpha}) = (\lambda)\frac{(a + 2b + c + e^\lambda * \mu_\alpha{}^\lambda)}{4}$$

$$AH(\acute{a}) = (\lambda)\frac{(a + 2b + c)(1 - v_\alpha)}{4}$$

Step 5: Rank the alternatives by the value. Make a sensitivity analysis with respect to the attitudinal character.
Step 6: End.

5.   A numerical example

In this section, first, we prove the effectiveness and rationality of the proposed TIARA model by using some practical problems in literature for product reliability assessment.

Let us consider a practical MAGDM problem originally reported by Wan et al., (2016). In this case, the original problem has been modified to make a new example, however using the same decision matrixes.

Suppose the product development team of a design company 'XZ' wants to redesign the crankshaft of a proposed new car. Since reliability information is scared at the early design phases. Hence, the failure information of an existing predecessor slewing gear is analyzed with the view to converting the information to appropriate design reliability knowledge. If four failure modes (A_1, A_2, A_3, and A_4) are identified by a group of Experts (E_1, E_2, E_3, and E_4) with weight vector $w = (0.2; 0.3; 0.35; 0.15)^T$ respectively, then the information they provided are quantitatively presented in Table 1-4.

Table 1. TIFN decision matrix by E1

A_i	C_1	C_2	C_3	C_4	C_5
A_1	([0.26,0.50,0.73]; 0.5,0.4)	([0.36,0.54,0.77]; 0.6,0.4)	([0.24,0.41,0.56]; 0.6,0.2)	([0.37,0.55,0.76]; 0.4,0.5)	([0.32,0.48,0.66]; 0.5,0.3)
A_2	([0.33,0.50,0.83]; 0.7,0.1)	([0.31,0.48,0.77]; 0.5,0.3)	([0.42,0.54,0.73]; 0.7,0.3)	([0.39,0.59,0.86]; 0.3,0.5)	([0.28,0.44,0.63]; 0.4,0.6)
A_3	([0.26,0.41,0.62]; 0.5,0.3)	([0.31,0.48,0.69]; 0.6,0.3)	([0.48,0.61,0.81]; 0.4,0.3)	([0.28,0.39,0.81]; 0.4,0.2)	([0.32,0.48,0.71]; 0.5,0.2)
A_4	([0.40,0.58,0.93]; 0.6,0.2)	([0.36,0.48,0.77]; 0.7,0.2)	([0.30,0.41,0.56]; 0.8,0.1)	([0.24,0.44,0.60]; 0.6,0.3)	([0.47,0.59,0.78]; 0.7,0.2)

Table 2. TIFN decision matrix by E2

A_i	C_1	C_2	C_3	C_4	C_5
A_1	([0.23,0.43,0.71]; 0.7,0.1)	([0.32,0.42,0.68]; 0.8,0.2)	([0.25,0.55,0.91]; 0.5,0.4)	([0.32,0.45,0.57]; 0.5,0.4)	([0.17,0.31,0.52]; 0.6,0.3)
A_2	([0.35,0.50,0.80]; 0.5,0.3)	([0.26,0.48,0.68]; 0.7,0.2)	([0.25,0.44,1.00]; 0.6,0.3)	([0.45,0.53,0.61]; 0.4,0.3)	([0.36,0.54,0.70]; 0.8,0.1)
A_3	([0.29,0.50,0.71]; 0.6,0.2)	([0.37,0.54,0.76]; 0.5,0.3)	([0.33,0.55,0.91]; 0.7,0.2)	([0.43,0.52,0.63]; 0.6,0.2)	([0.44,0.54,0.71]; 0.6,0.4)
A_4	([0.41,0.57,0.80]; 0.8,0.2)	([0.42,0.54,0.76]; 0.6,0.1)	([0.25,0.44,0.76]; 0.5,0.3)	([0.43,0.49,0.63]; 0.5,0.1)	([0.46,0.57,0.71]; 0.5,0.3)

Table 3. TIFN decision matrix by E3

A_i	C_1	C_2	C_3	C_4	C_5
A_1	([0.30,0.42,0.75]; 0.7,0.2)	([0.35,0.50,0.79]; 0.4,0.4)	([0.29,0.45,0.69]; 0.7,0.2)	([0.39,0.48,0.64]; 0.3,0.4)	([0.44,0.53,0.65]; 0.7,0.1)
A_2	([0.36,0.63,0.94]; 0.5,0.4)	([0.29,0.50,0.99]; 0.7,0.3)	([0.37,0.62,0.92]; 0.6,0.3)	([0.34,0.42,0.59]; 0.6,0.3)	([0.39,0.51,0.65]; 0.5,0.3)
A_3	([0.36,0.49,0.75]; 0.6,0.3)	([0.23,0.43,0.69]; 0.5,0.2)	([0.37,0.53,0.81]; 0.6,0.2)	([0.47,0.60,0.74]; 0.5,0.3)	([0.35,0.48,0.64]; 0.6,0.3)
A_4	([0.24,0.42,0.66]; 0.6,0.2)	([0.29,0.57,0.89]; 0.8,0.1)	([0.22,0.36,0.69]; 0.5,0.4)	([0.30,0.48,0.65]; 0.4,0.3)	([0.38,0.47,0.62]; 0.5,0.5)

Table 4. TIFN decision matrix by E4

A_i	C_1	C_2	C_3	C_4	C_5
A_1	([0.35,0.51,0.94]; 0.6,0.3)	([0.28,0.53,0.80]; 0.5,0.3)	([0.32,0.42,0.68]; 0.6,0.2)	([0.48,0.56,0.68]; 0.7,0.2)	([0.39,0.55,0.76]; 0.6,0.3)
A_2	([0.23,0.37,0.56]; 0.5,0.2)	([0.33,0.47,0.71]; 0.5,0.4)	([0.26,0.48,0.68]; 0.7,0.2)	([0.32,0.42,0.54]; 0.5,0.3)	([0.33,0.51,0.73]; 0.9,0.1)
A_3	([0.35,0.59,0.94]; 0.4,0.5)	([0.22,0.47,0.80]; 0.7,0.1)	([0.37,0.54,0.76]; 0.8,0.1)	([0.42,0.52,0.64]; 0.4,0.5)	([0.35,0.53,0.79]; 0.5,0.2)
A_4	([0.29,0.51,0.75]; 0.5,0.1)	([0.39,0.53,0.89]; 0.6,0.3)	([0.42,0.54,0.76]; 0.5,0.3)	([0.38,0.49,0.61]; 0.6,0.1)	([0.29,0.39,0.62]; 0.7,0.2)

The information are evaluated using the proposed algorithm when the criteria C_i; chance of failure (C_1), non-detection of failures (C_2), maintainability (C_3), economic safety (C_4), and economic cost (C_5) has the weight vector $\omega = (0.1848; 0.2217; 0.1617; 0.2100; 0.2217)^T$, respectively.

Following the algorithm of the proposed approach in Section 4, the failure mode alternatives are evaluated with respect to the criteria. Using the TIHFWG operator, the Expert's preference judgments $R^k (k = 1,2,3, ..., l)$ are aggregated to form the aggregated expert's decision matrix $R = (r_{ij})_{mxn}$, the result of the aggregation is shown in Table 5.

Table 5. Aggregated experts reliability information for the crankshaft

A_i	C_1	C_2	C_3	C_4	C_5
A_1	([0.28,0.45,0.76]; 0.64,0.23)	([0.33,0.49,0.75]; 0.55,0.33)	([0.27,0.46,0.70]; 0.60,0.27)	([0.38,0.50,0.65]; 0.42,0.40)	([0.31,0.44,0.62]; 0.61,0.24)
A_2	([0.33,0.52,0.81]; 0.53,0.29)	([0.29,0.49,0.80]; 0.62,0.29)	([0.32,0.52,0.86]; 0.63,0.30)	([0.38,0.48,0.63]; 0.45,0.35)	([0.35,0.50,0.67]; 0.60,0.30)
A_3	([0.31,0.49,0.73]; 0.54,0.31)	([0.28,0.48,0.73];0.55,0.24)	([0.38,0.55,0.83]; 0.61,0.21)	([0.41,0.52,0.70]; 0.49,0.29)	([0.37,0.53,0.70]; 0.56,0.30)
A_4	([0.32,0.51,0.76]; 0.64,0.19)	([0.35,0.54,0.82];0.68,0.15)	([0.27,0.42,0.69]; 0.55,0.30)	([0.33,0.48,0.63]; 0.49,0.22)	([0.40,0.51,0.68]; 0.56,0.35)

Using the I-TIHFWGA operator, when the weight vector associated with the criteria is given as; $\omega = (0.1848; 0.2217; 0.1617; 0.2100; 0.2217)^T$. The comprehensive evaluation for the four failure mode alternatives is achieved as shown in Table 6. For the different values of the parameter λ which is used to re-present and express the attitudinal character of the experts when evaluating and building reliability information for the new design product are given in Table 7.

Table 6. The comprehensive value for the failure modes (F)

F	Comprehensive value
A_1	([0.313, 0.468, 0.693]; 0.555, 0.298)
A_2	([0.332, 0.501, 0.743]; 0.563, 0.305)
A_3	([0.345, 0.510, 0.732]; 0.546, 0.271)
A_4	([0.337, 0.491, 0.714]; 0.582, 0.244)

Table 7. The ranking of all the failure modes with respect to the attitudinal parameter

λ	A_1		A_2		A_3		A_4		Ranking	F
	AS	AH	AS	AH	AS	AH	AS	AH		
0.1	0.075	0.034	0.078	0.036	0.078	0.038	0.077	0.038	$A_3 > A_2 > A_4 > A_1$	A_3
0.2	0.151	0.068	0.158	0.072	0.159	0.076	0.156	0.077	$A_3 > A_2 > A_4 > A_1$	A_3
0.3	0.231	0.102	0.241	0.108	0.242	0.115	0.239	0.115	$A_3 > A_2 > A_4 > A_1$	A_3
0.4	0.312	0.136	0.326	0.144	0.327	0.153	0.323	0.154	$A_3 > A_2 > A_4 > A_1$	A_3
0.5	0.396	0.171	0.414	0.180	0.414	0.191	0.411	0.192	$A_3 > A_2 > A_4 > A_1$	A_3
0.6	0.483	0.205	0.505	0.216	0.504	0.229	0.502	0.231	$A_2 > A_3 > A_4 > A_1$	A_2
0.7	0.573	0.239	0.599	0.253	0.597	0.267	0.597	0.269	$A_2 > A_4 > A_3 > A_1$	A_2
0.8	0.667	0.273	0.696	0.289	0.693	0.306	0.695	0.307	$A_2 > A_4 > A_1 > A_3$	A_2
0.9	0.763	0.307	0.797	0.325	0.792	0.344	0.797	0.346	$A_4 > A_2 > A_3 > A_1$	A_4

From the ranking result of the four failure modes assessment, the study can conclude therefore that failure modes A_2, A_3 and A_4 have the highest risk factors depending on the attitudinal character of the evaluating experts. The main advantage of the result presented here is the flexibility it provides in the decision-making process as well as, proves that the attitudinal character of the evaluating experts can indeed affect the final reliability decisions.

Comparison of Result: To prove the effectiveness of the model it is compared with the MAGDM approach which is based on the triangular intuitionistic fuzzy aggregation operator originally proposed by Li (2010) as shown in Table 8 and Table 9.

Table 8. The DMs preference information with the alternating the parameter

	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$
A_1	$Z(S_1, 0.1)$ $= 0.035$	$Z(S_1, 0.2)$ $= 0.071$	$Z(S_1, 0.3)$ $= 0.108$	$Z(S_1, 0.4)$ $= 0.147$	$Z(S_1, 0.5)$ $= 0.188$
A_2	$Z(S_1, 0.1)$ $= 0.039$	$Z(S_1, 0.2)$ $= 0.080$	$Z(S_1, 0.3)$ $= 0.123$	$Z(S_1, 0.4)$ $= 0.168$	$Z(S_1, 0.5)$ $= 0.215$
A_3	$Z(S_1, 0.1)$ $= 0.042$	$Z(S_1, 0.2)$ $= 0.086$	$Z(S_1, 0.3)$ $= 0.132$	$Z(S_1, 0.4)$ $= 0.180$	$Z(S_1, 0.5)$ $= 0.232$
A_4	$Z(S_1, 0.1)$ $= 0.040$	$Z(S_1, 0.2)$ $= 0.081$	$Z(S_1, 0.3)$ $= 0.125$	$Z(S_1, 0.4)$ $= 0.171$	$Z(S_1, 0.5)$ $= 0.219$

Table 9. The ranking of all the failure modes with respect to the attitudinal parameter

λ	Ranking	F
0.1	$A_3 > A_4 > A_2 > A_1$	A_3
0.2	$A_3 > A_4 > A_2 > A_1$	A_3
0.3	$A_3 > A_4 > A_2 > A_1$	A_3
0.4	$A_3 > A_4 > A_2 > A_1$	A_3
0.5	$A_3 > A_4 > A_2 > A_1$	A_3

From the comparison analysis, the study can conclude that the proposed approach is effective, feasible and rational. Since the results of the triangular intuitionistic fuzzy aggregation operator proposed by Li (2010) is in agreement with the results of the proposed model as shown in Table 10.

Table 10. The ranking of all the failure modes with respect to the attitudinal parameter

	Proposed Model		Model by Li (2010)	
λ	Ranking	F	Ranking	F
0.1	$A_3 > A_2 > A_4 > A_1$	A_3	$A_3 > A_4 > A_2 > A_1$	A_3
0.2	$A_3 > A_2 > A_4 > A_1$	A_3	$A_3 > A_4 > A_2 > A_1$	A_3
0.3	$A_3 > A_2 > A_4 > A_1$	A_3	$A_3 > A_4 > A_2 > A_1$	A_3
0.4	$A_3 > A_2 > A_4 > A_1$	A_3	$A_3 > A_4 > A_2 > A_1$	A_3
0.5	$A_3 > A_2 > A_4 > A_1$	A_3	$A_3 > A_4 > A_2 > A_1$	A_3

6. Conclusion

In this paper, a Triangular intuitionistic attitudinal ranking and aggregating (TIARA) model which is based on an Induced triangular intuitionistic hybrid fuzzy weighted geometric (I-TIHFWG) operator and an attitudinal rank score function has been proposed for reliability evaluation and prediction, at the conceptual design stage of the new product. The advantages of this model over currently existing ones are in its ability to account for the attitudinal character of design and reliability assessment

experts in the evaluation and prediction of the reliability of new products at the conceptual design stage and for decision-making. It helps in reducing the complexity in the product development process by representing holistically all the complexity and uncertainty using the Triangular intuitionistic fuzzy number (TIFN) which is a more generalized platform for expressing imprecise and inconsistent information and finally. It provides an opportunity for carrying out a sensitivity analysis using the attitudinal score function (attitudinal parameter), thereby addressing the ranking problem normally associated with the TIFN(s).

To demonstrate the effectiveness, feasibility, and rationality of the proposed model, it was applied for the evaluation of a hypothetical reliability assessment problem in literature and has been compared with the similar computational model. In the future, we will continue working on the application of the proposed model in other domain.

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